Online Multicasting Using Network Coding in Energy Constrained Wireless Ad hoc Networks

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Abstract—**We consider the optimized joint design of the physical, medium access control, and network layers to maximize the lifetime of energy constrained wireless ad hoc networks for online multicast applications. We consider two types of lifetime definitions, first node failure and multicast functionality. For each definition, we investigate two energy efficient approaches of minimum total energy and min-max per node energy usage to maximize network lifetime. Using network coding at the network layer, the problem of computing multipath flow for both schemes is formulated as a linear optimization problem. Simulation results are provided assessing the effectiveness of the algorithms.**

Keywords— linear optimization, cross-layer design, network coding, lifetime, online multicasting.

I. INTRODUCTION

Wireless ad hoc networks (WANETS) are composed of a collection of wirelessly communicating devices with no prearranged infrastructure. In such networks, the devices (nodes) are often powered by battery, and energy efficient operations are critical to prolong the lifetime of the connections. As an important application in WANETS, multicasting involves transferring data from a source node to a subset of nodes in the network. This paper addresses the problem of lifetime maximization for online multicasting in WANETS based on a cross layer approach, and considers unicast and broadcast as special cases of multicast.

Online multicasting refers to provisioning of a sequence of multicast requests on a per request basis. In this work, the provisioning involves scheduling in MAC layer, network coding (routing) in network layer and power control in physical layer. In fact, multicast requests are initiated at different nodes in the network over time. Each request is facilitated in a multicast session denoted by the quadruple $<s, D, r, \Omega>$, whose elements indicate the source, set of destination nodes, multicast rate and size of the data to be multicast, respectively. Obviously, these parameters may vary among different multicast sessions.

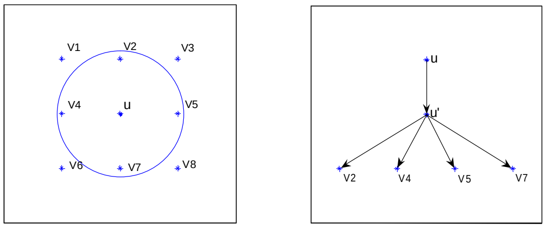
The term “lifetime” has numerous definitions. We consider two such definitions: time to first node failure [1] and multicast functionality [2]. The latter refers to the total time that the network provisions different multicast requests until it can no longer support a new multicast session. In order to maximize the lifetime, we consider two objectives of energy efficiency: (1) minimization of total energy consumption per request and (2) minimization of maximum energy consumption among nodes provisioning a multicast request.

For energy efficient online multicasting, researchers, e.g. in [1]-[3], use routing to find a multicast tree transferring data with a given size. Previously in [4] and [5], network coding is used instead of conventional routing to minimize power consumption or minimize energy per bit in multicasting. For maximizing lifetime in online multicasting, we set up a cross-layer design frame work based on network coding. The proposed solutions are obtained as linear programming formulations for online provisioning of multicast requests with a multicast rate guarantee.

The rest of this paper is organized as follows. Section II presents our system model and the problem of multicast network lifetime. In Section III, we formulate the proposed cross layer design solutions as four different algorithms. In Section V, we report on simulation experiments using the presented techniques.

II. PROBLEM FORMULATION

For the problem of lifetime-optimal multicast, we use “combined elementary graph” structure that is adapted from [4]-[5] and is comprised of multiple broadcast links. Each broadcast link modeled with an elementary graph G_k considers broadcasting of common information in the physical layer from one transmitter to a set of receivers. If $u \xrightarrow{c_k} Y_u = \{v_1, ..., v_n\}$ is the broadcast link in an elementary graph $G_k = (V_k, E_k, \boldsymbol{c}_k)$, it may be represented by a tree like graph with a virtual vertex u', where $V_k = \{u, u', v_1, ..., v_n\}$ is the set of nodes, $E_k = \{uu', u'v_1, ..., u'v_n\}$ is the set of edges and $\boldsymbol{c}_k = (c_k(uu'), c_k(u'v_1), ..., c_k(u'v_n))$ is the rate vector associated with the edges of the graph. Fig. 1 depicts an example of a broadcast link with its corresponding elementary graph. In fact, a virtual vertex plays the role of an artificial bottleneck, which constrains the rate of new information emanating



(a) (b)

Fig. 1.(a) node *u* is transmitting with power *p* and nodes v_2,v_4 ,v_5 and v_7 are in transmission range of broadcast link $u \xrightarrow{c_k} Y_u = \{v_2, v_4, v_5, v_7\}$ with threshold γ ,(b) elementary graph corresponding to broadcast link (a).

from a transmitter [5]. To construct all possible elementary graphs, an SNR-based approach is used.

The transmitter power, *p*, in the elementary graph is adjusted to facilitate a given minimum SNR, and hence rate $c = \frac{1}{2}\log(1+g)$, at the receiving nodes. For the broadcast link $u \xrightarrow{c_k} Y_u = \{v_1, ..., v_n\}$, we choose

$$c_k(uu') = c_k(u'v_1) = ... = c_k(u'v_n) = c \qquad (1)$$

By adjusting transmission power of each node we construct all possible elementary graphs with new receivers that fall in the transmission range of the transmitting node. If the total session time is denoted by *t*, the combined elementary graph is constructed by timesharing among a finite set of elementary graphs such that

$$\sum_k l_k \le t \qquad (2)$$

where l_k is the relative share of time for the graph $G_k = (V_k, E_k, \boldsymbol{c}_k)$. We denote the combined elementary graph of the network with $G = (V, E, \boldsymbol{c})$; where $V = \mathbf{U}_k V_k$ is the set of nodes, $E = \mathbf{U}_k E_k$ the set of links, and $\boldsymbol{c} = \sum_k \boldsymbol{c}'_k$ the capacity vector associated with the edges. The vector $\boldsymbol{c}'_k$ of length $|\mathbf{U}_k E_k|$ is composed of elements of $\boldsymbol{c}_k$ for the corresponding branches of the elementary graph G_k and zero elsewhere. We have $G = \sum_k l_k G_k$. We consider a flow vector $\boldsymbol{f_d}$ of length |*E*|, with elements $f_d(vw)$ $\forall vw \in E$,as the number of bits carried by the edge, during the session time *t*, for destination ***d***.

In the lifetime maximization problem, the goal is to find the timeshare l_k during which the transmitter of each elementary graph operates for each session. This in turn determines the traffic on each edge and, how the information flow is coordinated within the network. For maximizing the network life time, we consider interference free communication model in the sense that at each time instance only one node may transmit. This in general may be extended to the case where more than one simultaneous transmission could take place only if they are sufficiently distanced from each other. Although, the latter may help increase the rate, however, is still a sub-optimum approach from an energy preservation point of view [5]. Furthermore, we consider the case in which all nodes are of equal importance; and that different multicast sessions are subsequently administered.

III. ONLINE LIFETIME MAXIMIZATION ALGORITHMS USING NETWORK CODING

We are interested to find, in general, a multipath for each multicast request such that total amount of data transfer trough the network is maximized. For this problem, using network coding, we establish an optimal polynomial-time solution that is based on linear programming. We consider common capacity $\boldsymbol{c}_k$, for all elementary graphs. In the proposed formulation, for *l*th session, flow values are determined by assigning a timeshare of l_k^l to each broadcast link with capacity $\boldsymbol{c}_k$ so that a total flow Ω^l is transmitted from the source to the destinations. The multicast rate, r^l , is defined as the number of bits of common information delivered from source to all destinations in unit time. We define the total data transfer as $\sum_l \Omega^l$ and lifetime as the total duration of provisioning multicast requests $\sum_l t^l$, where $t^l = \Omega^l / r^l$. The goal is to maximize $\sum_l t^l$ for data transfer through network for different requests. In the followings, we present two algorithms for online multicast provisioning in wireless ad hoc networks.

A. Minimum total energy multicast using network coding

In this section, we aim at constructing the minimum total energy multipath for each request with multicast rate guarantee that is demanded by the session. Therefore, the objective is to minimize $P(\sum_{k:G_k} l_k^l G_k) = \sum_{k:G_k} l_k^l p_k(G_k)$ and is to be fulfilled under the scheduling and flow constraints.

Proposition 1: The *Minimum total Energy-First node Failure* (*MEFF*) problem for provisioning of the *l*th multicast session $< s^l, D^l, r^l, \Omega^l >$, $s^l \in V$ and $D^l \subset V$ over a WANET with a set of broadcast links as elementary graphs $G_k = (V_k, E_k, \boldsymbol{c}_k)$, is stated as follows.

$$\text{Min} \sum_{k:G_k} l_k^l p_k$$

Subject to

$$C.I. \begin{cases} \sum_k l_k^l \le t^l, \\ l_k^l \ge 0 \qquad \forall k \end{cases}$$

$$C.II. \begin{cases} \sum_{w \in V: sw \in E} f_d^l(s^l w) - \sum_{u \in V: us \in E} f_d^l(us^l) = r^l\ t^l \qquad \forall d \in D^l \\ \sum_{w \in V: vw \in E} f_d^l(vw) - \sum_{u \in V: uv \in E} f_d^l(uv) = 0 \quad \forall v \in V - \{s^l, D^l\}, \forall d \in D^l \\ \sum_{w \in V: dw \in E} f_d^l(dw) - \sum_{u \in V: ud \in E} f_d^l(ud) = -r^l\ t^l \qquad \forall d \in D^l \\ 0 \le f_d^l(vw) \le l_k^l c_k(vw) \qquad \forall\ vw \in E_k, \forall d^l \in D^l \end{cases}$$

(3)

In this problem, λ is the control variable (f depends on λ) that is found to minimize total energy consumption of the lth session. As evident, the objective function and constraints are linear equations. Therefore, the presented optimization problem is an explicit linear program and will return the optimal solution of (3). The constraints are elaborated below.

C.I. In link scheduling, the total time that is shared between broadcast links, $\sum_k l_k^l$, must be smaller than or equal to the session time, t^l.

C.II. This set of constraints is imposed by the flow conservation equations. In contrast to common flow conservation formulations in network problems that are solved with routing, we use network coding in our flow formulation. The maximum flow on edge *vw* must be smaller than the capability of that link for transferring data. The last equation states the capability of each edge in transferring data as a function of the allocated timeshare l_k^l, and its transmission rate $c_k(vw)$. If there is a feasible flow assignment, Max-Flow-Min-Cut is fulfilled and therefore, a network code exists [5]. Note that the third equation is implied from the first two equations, and therefore, can be eliminated.

In the proposed *MEFF* formulation, if at least one node is depleted of energy during a session, the network life time ends. A better approach is to use an alternate definition of lifetime; instead of first node failure, one may opt for the time that network may still support a new multicast session. This is considered by imposing another constraint *C.III* to the *MEFF* problem, hence referred to as the *Minimizing total Energy-Multicast Functionality* (*MEMF*) problem. This constraint ensures that a minimum energy multipath multicast graph is obtained in which the amount of energy consumed by each node during each session is smaller than its initial energy.

Proposition 2: The *Minimum total Energy-Multicast Functionality* (*MEMM*) problem for provisioning of the lth multicast session $< s^l, D^l, r^l, \Omega^l >$, $s^l \in V$ and $D^l \subset V$ over a WANET with a set of broadcast links as elementary graphs $G_k = (V_k, E_k, \boldsymbol{c}_k)$, is stated as follows.

$$\text{Min} \sum_{k:G_k} l_k^l p_k$$

Subject to

$$C.I. \begin{cases} \sum_k l_k^l \le t^l, & \\ l_k^l \ge 0 & \forall k \end{cases}$$

$$C.II. \begin{cases} \sum_{w\in V:sw\in E} f_d^l(s^l w) - \sum_{u\in V:us\in E} f_d^l(us^l) = r^l\ t^l & \forall d\in D^l \\ \sum_{w\in V:vw\in E} f_d^l(vw) - \sum_{u\in V:uv\in E} f_d^l(uv) = 0 & \forall v\in V-\{s^l, D^l\}, \forall d\in D^l \\ \sum_{w\in V:dw\in E} f_d^l(dw) - \sum_{u\in V:ud\in E} f_d^l(ud) = -r^l\ t^l & \forall d\in D^l \\ 0\le f_d^l(vw) \le l_k^l c_k(vw) & \forall\ vw\in E_k, \forall d\in D^l \end{cases}$$

$$C.III. \begin{cases} \sum_{k:G_k\in G_{k|v}} l_k^l p_k < e_v^l & v\in V \\ G_{k|v} = \{\ G_k\ \mid \text{transmitter of } G_k\ is\ v\in V\} & \\ e_v^l :\ \text{energy of node } v\in V \text{ when } l\text{th session starts} & \end{cases} \tag{4}$$

B. Minimizing maximum energy usage per node in multicasting using network coding

In the previous algorithm, we find the multipath with minimum total energy. Minimum total energy consumption does not guarantee maximum lifetime for a network, as has been noted in [7]-[8]. Therefore, we consider an alternate objective function that has an inherent meaning of lifetime and try to maximize the lifetime of each node. We aim at minimizing the maximum energy usage of each node through a Min Max algorithm.

Proposition 3: The *Minimizing Maximum energy usage-First node Failure* (*MMFF*) problem for provisioning of the lth multicast session $< s^l, D^l, r^l, \Omega^l >$, $s^l \in V$ and $D^l \subset V$ over a WANET with a set of broadcast links as elementary graphs $G_k = (V_k, E_k, \boldsymbol{c}_k)$, is stated as follows.

$$\text{Min}\ \underset{v}{\text{Max}} \sum_{k:G_k\in G_{k|v}} l_k^l p_k$$

Subject to

$$C.I. \begin{cases} \sum_k l_k^l \le t^l, & \\ l_k^l \ge 0 & \forall k \end{cases}$$

$$C.II. \begin{cases} \sum_{w\in V:sw\in E} f_d^l(s^l w) - \sum_{u\in V:us\in E} f_d^l(us^l) = r^l\ t^l & \forall d\in D^l \\ \sum_{w\in V:vw\in E} f_d^l(vw) - \sum_{u\in V:uv\in E} f_d^l(uv) = 0 & \forall v\in V-\{s^l, D^l\}, \forall d\in D^l \\ \sum_{w\in V:dw\in E} f_d^l(dw) - \sum_{u\in V:ud\in E} f_d^l(ud) = -r^l\ t^l & \forall d\in D^l \\ 0\le f_d^l(vw) \le l_k^l c_k(vw) & \forall\ vw\in E_k, \forall d\in D^l \end{cases} \tag{5}$$

In this problem, λ is the control variable (f depends on λ) that is found to minimize the total energy consumption of the lth session. The presented optimization problem is an explicit linear program and will return the optimal solution of (5). The *C.I.* and *C.II.* constraints are basically those elaborated for the *MEFF* algorithm .

The *MMFF* algorithm finds the node with maximum transmission energy and aims at minimizing its energy usage. However, other nodes may not necessarily have their

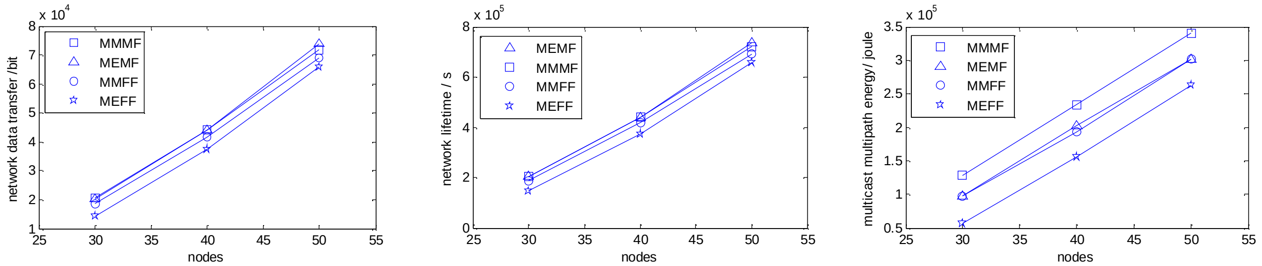


Fig. 2. Mean of network throughput, network lifetime and energy consumption over 100 random networks.

minimum energy consumption. Therefore, the total energy usage with the *MMFF* algorithm is expected to be more than the *MEFF* algorithm.

As discussed, we may still consider constraint *C.III* with the *MMEF* formulation; leading to an alternate scheme referred to as *Minimizing Maximum energy usage-Multicast Functionality* (*MMMF*), described below.

Proposition 4: The *Minimizing Maximum energy usage-Multicast Functionality* (*MMMF*) problem for provisioning of the *l*th multicast session $< s^l, D^l, r^l, \Omega^l >$, $s^l \in V$ and $D^l \subset V$ over a WANET with a set of broadcast links as elementary graphs $G_k = (V_k, E_k, \boldsymbol{c}_k)$, is stated as follows.

$$\text{Min}\ \underset{v}{\text{Max}} \sum_{k:G_k \in G_{k|v}} l^l_k p_k$$

Subject to

$$C.I.\ \begin{cases} \sum_k l^l_k \le t^l, \\ l^l_k \ge 0 \qquad\qquad \forall k \end{cases}$$

$$C.II.\ \begin{cases} \sum_{w \in V: sw \in E} f^l_d(s^l w) \ - \sum_{u \in V: us \in E} f^l_d(us^l) \ = \ r^l\ t^l \qquad \forall d \in D^l \\ \sum_{w \in V: vw \in E} f^l_d(vw) \ - \sum_{u \in V: uv \in E} f^l_d(uv) = 0 \ \ \forall v \in V - \{s^l, D^l\}, \forall d \in D^l \\ \sum_{w \in V: dw \in E} f^l_d(dw) \ - \sum_{u \in V: ud \in E} f^l_d(ud) = \ \text{-}r^l\ t^l \qquad \forall d \in D^l \\ 0 \le\ f^l_d(vw)\ \le\ l^l_k c_k(vw) \qquad \forall\ vw \in E_k, \forall d \in D^l \end{cases}$$

$$C.III.\ \begin{cases} \sum_{k:G_k \in G_{k|v}} l^l_k p_k < e^l_v \qquad v \in V \\ G_{k|v}\ = \{\ G_k\ \ |\ \text{transmitter of}\ G_k\ \ is\ v \in V\} \\ e^l_v :\ \text{energy of node}\ v \in V\ \text{when}\ l\text{th session starts} \end{cases} \tag{6}$$

IV. PERFORMANCE EVALUATION

We consider networks with different node densities distributed randomly in a $80 \times 80\ m^2$ region. The transmission rate of each link $\boldsymbol{c}_k$ is set to 1 bit per seconds. A sequence of random multicast requests is generated as sessions $< s^l, D^l, r^l, \Omega^l >$, in which the source s and destination set D are determined randomly within the network. Also for each request, the multicast data size and rate are chosen randomly from a uniform distribution between [1 10] bit and [0.1 0.2] bit per second, respectively. Three network sizes of 30, 40 and 50 nodes are considered; for each size 100 random networks are generated. Each algorithm is evaluated over these 300 networks with the same sequence of sessions.

Fig. 2 shows that *MEMF* and *MMMF* algorithms with the multicast functionality constraint *C.III* provide much longer network lifetime than *MEFF* and *MMFF* algorithms that consider a first node failure definition of lifetime. The former schemes facilitate the communication from source to destinations through other nodes, even when a first node runs out of energy. Since the *MEMF* algorithm consumes the least amount of energy on average for each session, it also facilitates the largest network throughput during the network lifespan.

The Min-Max algorithms minimize the energy usage of the node with maximum consumption, and not that of all other nodes. Therefore, for one session the overall energy consumption of minimum total energy algorithms is less than that of the min-max algorithms. Still, under the first node failure definition for lifetime the proposed Min-Max algorithm *MMFF* provides better network throughput than the minimum total energy algorithm *MEFF*. Since in minimum total energy algorithms, the depletion rate of nodes' energy is not equal, therefore, a first node is depleted of energy faster, when compared to the min-max algorithm.

V. CONCLUSIONS

Different energy efficiency objectives and lifetime definitions have been considered in the literature for wireless ad hoc networks. In this paper, we consider network coding and assess this variety in a common network setting. We present solutions based on both minimum total energy and min-max algorithms. As the results indicate, considering network lifetime as multicast functionality as opposed to first node failure in the design, improves the WANET lifetime. In online applications that requests arrive over time, *MMFF* outperforms *MEFF*, as min-max algorithm has a better notion of lifetime. The *MEMF* algorithm provides the best network throughput as it consumes the minimum energy for each session.

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